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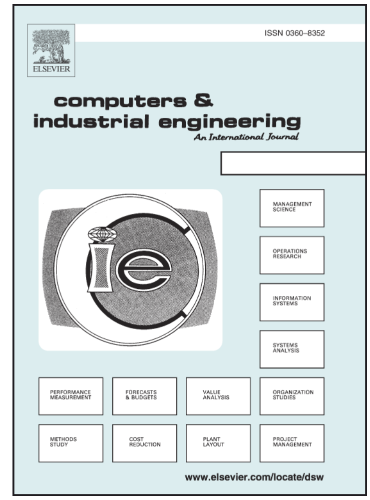

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Enhancing supply chain production-marketing planning with geometric multivariate demand function (a case study of textile industry)

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Highlights

- A joint model of multi-site production and marketing is established.
- Revenue management concept in aggregate production planning problems is considered.
- A new decreasing power function with leakage rate is developed.
- Generalized geometric programming approach is tailored in the solution algorithm.
- Applicability of the proposed framework is studied in a garment supply chain.

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Abstract

In this paper, a multi-period, multi-product, multi-site, multi-sales channel aggregate production planning problem including ordering preferences is presented in an integrated two-echelon supply chain to avoid the sub-optimality caused by separate, sequential decisions of production and the marketing/retailing chain. Each customer demand class is affected by price, marketing expenditures and product quality involving customer willingness-to-pay. In addition, the immigration of customers between submarkets (i.e. cannibalization) is considered in the market-segmented environment due to imperfect segmentation. This research develops a geometric programming model to formulate the issue of joint price differentiation and multi-site aggregate production planning decisions by maximizing the total profit of the supply chain. To tackle the model and obtain solutions, we tailor an efficient analytical solution procedure to convert the original highly non-linear programming model into a convex programming equivalent. Finally, a numerical study of garment supply chain is presented to demonstrate the performance of the model and solution approaches. The research findings indicate a positive relationship between the scaling constant of price-dependent demand and the total profit rate. Moreover, as price gaps grow, the utility of price differentiation is decreased.

Keywords: Multi-site production planning, Integrated production-marketing, Price differentiation, Geometric programming, Non-linear programming.

1. Introduction

Aggregate production planning (APP) has been traditionally receiving a lot of attention as a leading operation planning technique. The APP is to aggregate all production information in a medium term planning horizon to meet demand swings using shared physical resources. Supply chain responsiveness and excellency may thus be enhanced through efficient implementation of APP.

Evidently, intra-organizational supply chain management requires cross-functional integration within a firm. For the functional areas of the firm, marketing and production sub-plans constitute the most important features of the corporate plan. These plans require many specific decisions in order to meet corporate financial growth, market share and other objectives. Conventionally, production and manufacturing plan is guided by the marketing planning with the realized demand. The function of marketing plan is to specify the type of products to be offered, its sales channels and its price, while production and manufacturing plan determines how to efficiently utilize and allocate corporate resources to deliver the marketing. Instead of using a coordinated strategy, these plans are often developed sequentially/individually in different departments. For further clarification, consider a market with fierce competition such as the apparel industry. The marketing department is willing to use a wide range of instruments (e.g. pricing and marketing expenditures) to stimulate consumer demands or set the customer needs and wants in different sales channels. In this regard, the production department, to meet demand fluctuations, then determines production, inventory and workforce levels over the planning horizon. As Berry et al. (1999) stress, the debate of the linkage between manufacturing and marketing are crucial whenever an enterprise tends to be competitive in the target market.

It is worthwhile to note that practitioners suffer from the lack of coordination between marketing and production functions. By reviewing marketing–operations interface models, Tang (2010) argued the significant importance of coordination between the internal-focused functional area (i.e. operation and production) and the external-focused function (i.e. marketing)[2]. Piercy (2010) stresses that marketing and operations areas are interdependent. In this regard, cross-functional collaboration is crucial for the proper alignment of marketing and operational plans. This collaboration has been known as a prerequisite for effective firm performance (Piercy and Ellinger, 2015). To be specific, Hess and Lucas (2004) studied *Sport Obermeyer Company* (an apparel producer with several stores throughout the US) that faces the integrated decision between its marketing and production plans. More surprisingly, Berry et al. (1999) exemplified an apparel company located in Thailand (named *Anonke Apparel*) to illustrate the importance of the coordination of marketing/manufacturing strategy. *Anonke Apparel* operates in multiple demand classes such as our case study. This company tries to overcome "the inability of marketing and manufacturing to jointly develop consistent strategies" (Berry et al. 1999), despite massive investments in manufacturing. Fisher et al. (1994) studied *The L.L. Bean Company* (a waterproof boot and clothing producer) for its challenges related to inventory planning and market demands. At *Clothco* (an internet-based clothing retailer), the misalignment between operations and marketing departments led to high operational costs (Piercy, 2010). In addition, there exist several practical real-world examples in the joint production-marketing literature (e.g. Tang et al., 2009; Tang, 2010; Chan et al., 2004). More recently, Lamas and Chevalier (2018) noted companies are increasingly adopting the sales and operation planning.

Moreover, the constant demand assumption is unrealistic for most retailing industries, especially at the operational level of supply chains. Indeed, one of the key operational issues is to deal with the market-dependent issue in demand estimation. Incorporating demand-dependent issues will not only improve the realism of the APP problem, but is also likely to generate new and relevant insights. In real life, the demand rate for a specific item, especially for textile products, can be affected by many parameters such as the selling price, advertisement and other marketing parameters.

Differential pricing in market-segmented situations has long been acknowledged as a profitable pricing policy (Philips, 1989). It has been reported by many research studies (see Philips, 2005; Zhang et al., 2010; Braouezec, 2012; Raza, 2015) that there are numerous examples in which a firm persuades or manipulates its customers to different channels using differentiated prices as a strategy to increase the profits when consumers' tastes and valuations

[2]The interested reader may refer to Figure A.1 for a conceptual model for coordination between production and marketing.

of an item differ. As Ghasemy Yaghin et al. (2015) mentioned, perfect market segmentation is usually impossible for the practitioners. That is, it is unrealistic to imagine that this total potential gain could easily be captured, since immigration to pay the lower price acts as a powerful incentive for customers in high-priced segments. Notably, this switching behavior, which is also referred to as cannibalization, allows some degree of demand leakage from the high-priced market segments to the low-priced segments. Some recent studies have taken into consideration the demand leakage effect on joint pricing and inventory control. These studies include Zhang et al. (2010) and Raza (2015).

Due to the demand fluctuations and resource restrictions, cost-efficient production planning is an important problem over a midterm planning horizon in many industries such as the garment industry. Optimizing capacity utilization is extremely difficult in this labour-intensive industry with demand dependent on the trio of price, advertisement, and quality. The situation gets even harder if there is a two-echelon supply chain, one of which consists of multiple parallel manufacturing sites. Over a midterm planning horizon, the workforce levels of regular time and overtime, the numbers of workers to be hired and laid off in each site at each period, and the inventory levels of raw material and finished product to be carried for each period should be determined to meet the variable demand. The central problem here is to optimally coordinate the demand-related decisions (price-setting and marketing expenditure determination) and supply-side decisions (cost-efficient production and ordering planning).

The newly considered assumptions of this paper can be attributed to the realization of several industrial APP situations namely textile supply chains. As Leung et al. (2003) mention, a multinational textile company with its manufacturing factories located in Asian and European countries is an instance of this kind of apparel industry. Moreover, ordering preferences are taken into account to supply the textile products from credible manufacturers. The retailing planning department collects orders and faces multiple distribution channels (e.g., offline and online stores). In the presence of relative substitutability of apparel products in the market, demand for these items is highly dependent on price and quality. Therefore, whenever the price of these products in a sales channel is high, customers attempt to find a way to immigrate to other sales channels. Lastly, marketing expenditures are generally spent on textile items to amplify demand and avoid extended storage times, especially for fashionable ones.

In summary, the research questions (RQ) that this study aims to address are:

- RQ1: How are the marketing and revenue management (RM) practices incorporated into the aggregate production planning issues with the presence of multivariate customer demand response curve? What is the appropriate integrated policy of production and marketing over the tactical horizon in order to determine order and production quantities as well as market-dependent variables?
- RQ2: How do the marketing aspects influence the total profit? To that end, the research is going to investigate the impact of marketing expenditures and pricing parameters on the total profit.
- RQ3: Will the total profit be affected by the leakage rate between market segments? What are the roles of price differentiation and demand leakage rate in the multi-channel retailing of the two-echelon supply chain?

The main purpose of this study is to develop an APP model through the lens of revenue management where the demand rate depends on the selling price, marketing expenditures and quality in a market-segmented environment. To answer these questions, we develop a novel mathematical programming model to jointly determine marketing and production plan over the tactical planning horizon. In the proposed model, the demand rate is assumed to be a decreasing continuous power function of the unit selling price, increasing function of marketing efforts and quality of the products. The function is considered in the presence of leakage behavior when the retailer faces multiple demand classes in a two-echelon supply chain. Logically, the retailer is willing to order its needs as much as possible from credible manufacturers (i.e. ordering preferences). Besides, the production echelon consists of multiple (parallel) manufacturing sites that are connected to a multi-sales channel retailing echelon. Based on these assumptions, a geometric mathematical model is formulated, and a novel solution methodology is developed to maximize the total profit of the supply chain.

To summarize, the main contributions of this research work are as follows.

- From a practical point of view,
    1) We identified that products' prices, marketing expenditures and products' quality are the major drivers of customers demand in many industries[3]. Hence, we take these features into account in the customer response curve modelling. As a consequence, a novel, comprehensive demand function is proposed to take customer behavior into consideration. In detail, this demand is a decreasing power function of the unit selling price, increasing function of marketing efforts and the quality of the products in the presence of leakage behavior.

This aspect of our contribution addresses part (1) of the first research question (i.e. RQ1) and RQ2. The research finding indicates a positive relationship between the scaling constant of price-dependent demand and total profit rate. Furthermore, marketing activities and product quality perform active roles in the total profit of the supply chain. Our numerical results also show that the higher marketing budget leads to more profitability in comparison with the fixed overall marketing investment.

2) The need of multi-site production, as a parallel manufacturing system, has been heavily emphasized to meet demand fluctuations in several industries[4]. Hence, a multi-period, multi-product, multi-site, and multi-sales channel aggregate production-marketing planning problem with ordering preferences is presented in a two-echelon supply chain, considering imperfect market segmentation, to avoid the sub-optimality caused by separate, sequential decisions of production and marketing/retailing chain.

Part (2) of RQ1 is investigated via this facet of our contribution. We build an integrated multi-site production-marketing mathematical model. Through this, our numerical result shows that production sites actively try to meet an imperfect market-segmented consumer demand over the tactical planning horizon.

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[3]For instance, the nature of the newly developed products indicates that demand is highly dependent on the aforementioned drivers. Meanwhile, Simchi-levi et al. (2008) noted that product quality of apparel products and prices are important to profitability through investigating of *Zara Company*.

[4]Automotive (e.g. Bullinger et al., 1997; Gnoni et al., 2003; Torabi and Hassini, 2009), LCD (Chen et al., 2009), textile (e.g. Leung et al., 2003; Leung et al., 2006) manufacturing industries are notable instances.

3) Unlike the vast majority of APP problems studied in the literature, this research investigates a multiple sales channel problem that is widely observed in the market sector. Therefore, we develop a decision tool that can assist supply chain managers to better understand the effect of price differentiation with leakage behavior on the joint optimal production, pricing and ordering policies in APP-related decisions with product quality and ordering preferences. Considering the effects of product quality on end consumer demand is the other factor that differentiates this paper from the previous ones existing in the literature. Finally, a real case study in the clothing supply chain is studied to show the model applicability and effectiveness of the solution procedure.

RQ3 is addressed by this aspect of contribution. We found that as price gaps grow, the utility of price differentiation is decreased. In other words, there is an inverse relationship between the leakage rate and supply chain profitability. This finding is consistent with that of Zhang et al. (2010) and Ghasemy Yaghin (2018).

- From a theoretical point of view,
  - A new non-linear optimization model is formulated to determine the production-marketing planning under consideration. It includes power functions and non-convex terms in the objective function and constraints.
  - A novel solution procedure regarding generalized geometric programming is involved to tackle the resultant non-linear programming (NLP) model. The procedure utilizes a convexification strategy and transforms non-convex signomial terms to convexified counterparts. In other words, an analytical algorithm is tailored to search the solution space in an effort to determine the optimal decision variables.

The remainder of this paper is organized as follows. We first review the literature related to the focus of our study in Section 2. Section 3 mathematically models the supply chain production planning problem with price differentiation and ordering preferences. The solution algorithm based on the geometric programming approach is presented in Section 4. Section 5 provides an application of the proposed framework in a garment supply chain. Finally, suggestions and concluding remarks are provided in Section 6.

2. An overview of literature

2.1. APP in supply chains

Over the past decade, there has been an increased interest in APP modeling situations that involve more than one element of a supply chain. Kanyalkar and Adil (2007) studied time and capacity aggregated multi-site production planning, a detailed production planning and a detailed distribution planning concurrently and formulated the problem by a mixed integer linear goal programming model (MILGP) to make an integrated optimal supply chain plan. Torabi and Hassini (2008) integrated distribution, production, and procurement planning into a single model which also included multiple suppliers, one manufacturer and multiple distribution centers. The critical parameters were in uncertainty and conflicting objectives were considered simultaneously for a supply chain master planning model. In a multi-site logistics system, Kanyalkar and Adil (2010) further addressed integrated detailed production, procurement and distribution plans in which a countrywide aggregate production plan is integrated with a detailed plan. They developed a robust optimization model considering model robustness and solution robustness in the objective function for integrated planning in three dimensions. In a supply chain under uncertainty, Al-e-hashem et al. (2011) presented a multiple objective programming model for multi-site APP considering customer satisfaction. In a similar line, Leung et al. (2006) analyzed the multi-site APP problem by demand uncertainty. In a multi-national environment, Leung and Chan (2009) studied the APP problem with resource utilization consideration in a surface and materials science company. Also, Fahimnia et al. (2013) reviewed the state of the art in integrated production–distribution planning models and classified the literature into seven categories based on the degree of complexity.

More recently, Gholamian et al. (2016) developed a mathematical model for APP in a supply chain under demand uncertainty by a fuzzy multi-objective optimization method. The multiple objectives were total loss, customer satisfaction, labor productivity, and variability in a multi-product, multi-site and multi-period supply chain network. Considering collection and recycling centers, Entezaminia et al. (2016) proposed a multi-objective model for a multi-site APP problem in a green supply chain in order to incorporate the profit and green principles in an APP problem. They made a distinction between products in terms of environmental criteria such as recyclability and ease of disassembly, biodegradability, energy consumption and product risk quantified by AHP methodology. Sarkar (2013) also addressed the production-inventory problem for a deteriorating item in a two-echelon supply chain with transportation consideration. There are also lots of papers to deal with the production-distribution problem (see Torabi and Moghaddam, 2012; Fahimnia et al., 2013; Raa et al., 2013; Su et al., 2015; Zheng et al., 2016 and references therein).

Quality improvement is taken into consideration by Kim and Sarkar (2017) in a joint replenishment problem to clean a complex multi-stage production system from defective items. They proposed a stochastic inventory model to determine replenishment and shipment policies along with the quality factor. Considering the inspection of the product's quality, Sarkar (2016) addressed a supply chain coordination problem in a two-echelon setting to reduce the joint total cost among supply chain members. Some research works studied the quality of the returned products in reverse logistics and sustainable supply chain problems. For instance, Guo and Ya (2015) studied a manufacturing/remanufacturing system in which the recycling rates, buyback cost and remanufacturing cost are dependent on the different quality level of recycled products. Pariazar and Sir (2018) investigated the impact of disruptions on product quality and availability in integrated strategic and tactical supply chain planning problems. Sarkar et al. (2017) incorporated the quality dependent price of the returned product into a multi-echelon integrated manufacturing system with a third party logistics provider.

2.2. Marketing and RM in APP studies

In the real competitive world, the practice of allowing pricing and RM is increasing, and the relevant midterm tactical planning is becoming more interconnected with retailing, marketing and distribution activities. In this vein, few operations research-based works are applied to consider the RM practice with APP problems and therefore, the body of literature is very scarce in this regard. Aucamp (1986) incorporated marketing issues into the basic production planning model. Feiring and Mak (1995) developed a closed-loop procedure to link APP with marketing pricing for single item environment and dynamic demand. Besides, Yenradee and Sarvi (2007) analyzed the relationship between the price of the product and the amount of demand, and developed an integrated APP and pricing to determine suitable prices of the product in each period with elastic demand. Yenradee and Piyamanothorn (2011) investigated the integrated model of aggregate production planning and marketing promotion planning for a real case of a consumer product factory in Thailand. They provided a general optimization model that can help make decisions in marketing promotion in accordance with production planning to maximize the profit for the company. Martínez-Costa et al. (2013) reviewed the aggregate production and marketing planning models from the mathematical optimization point of view. Remarkably, the only research studies that include RM practices in APP literature are those of Ghasemy Yaghin et al. (2012) and Ghasemy Yaghin (2018). Ghasemy Yaghin et al. (2012) took into account a multi-period discount problem for products that undergo several price cuts over time. In their approach, the prices gradually decrease, as newer, more advanced products replace them in the market such that a retailer charged a single price to different customers for exactly the same good. Namely, more elastic consumers are charged with lower prices, and less elastic consumers are charged with higher prices. This can lead to "unfair" outcomes and additional profit can be extracted from a marketplace by tailoring different prices (Raza, 2015). Ghasemy Yaghin (2018) addressed the APP problem with multiple demand classes by non-linear optimization formulation. He considered consumer demand in each market as a function of advertisement and selling price.

On the other hand, as a continuance of Ghasemy Yaghin (2018)'s paper using a geometric programming solution procedure, we develop a multivariate demand response curve depending on quality issues to formulate the consumer behavior. Moreover, we take backorder level and ordering preferences of an integrated supply chain production-marketing problem into consideration in a parallel logistics system.

Features of the publications surveyed in this section are summarized in Table 1. It is apparent that no previous APP model has simultaneously considered differential pricing and multiple sales channels of retailing echelon, especially with leakage behavior, price and quality-dependent demand with credible sources of order. Combining these elements, this paper presents a geometric programming model in which the consumer demand depends on the selling price, marketing expenditures and quality as a decreasing power function.

Table 1 to be inserted here

Noteworthy, in the column of optimization approach, LP and MONLP mean linear programming and multi-objective non-linear programming, respectively. MOLP denotes multi-objective linear programming. Moreover, GP is an abbreviation of geometric programming.

3. Mathematical formulation

3.1. Problem description

The problem can be described as follows: consider a two-echelon supply chain including n_s production sites (PSs) manufacturing n_g product families for a retailer with n_{ms} sales channels to meet the consumer demand. The retailing echelon is willing to acquire the items from the parallel plants where they are produced, and this acquisition causes an inventory ordering cost and also a holding cost. Moreover, retailing echelon, operating in a monopoly market, is involved in storing and selling n_g finished products to different classes of customers in n_{ms} pre-determined market segments (MSs). Additionally, the demand response function is really affected by price, product quality and marketing expenditures. A set of production sites is to deliver orders to manufacture different types of products by involving some issues related to the plant operation and labors over T_{ph} planning horizon. Also, some of the manufacturing sites are credible with the good reputation. The plants are capacitated and must supply the retailer's order. Granted that the aforementioned companies in a two-echelon supply chain system are connected and interrelated with each other. Our objective is to find the best planning decisions over a multi-period mid-term horizon, in an integrated and coordinated manner. In other words, the two-echelon supply chain is under centralized control and supply chain information from each of the echelons is available to the decision maker.

3.2. Notation and model development

Now, we are going to describe all the components of the model. Consider the following sets: plants *J*, sales channels n_{ms}, product families n_g, planning horizon T_{ph} and credible manufacturers *BQ*.

3.2.1. Parameters

The parameters of the model are the costs, inventory levels, labor level available and demand related ones. Following are the input parameters used in the model.

Demand and marketing parameters

v_{kit}	scaling constant of price-dependent demand term of sale channel *k* for product *i* at period *t*
α_{kit}	price elasticity to demand term of sale channel *k* for product *i* at period *t*
γ_k	elasticity of demand with respect to expenditures in market segment *k*
θ_{it}	quality elasticity of demand of product *i* in period *t*
BUD^{max}	the maximum marketing cost ($)
LR	leakage fraction between MS *k* and the n_{ms}-th MS

Parameters related to production and inventory costs

c_{ij}^{rt}	unit manufacturing cost of product *i* in PS *j* at regular time ($/unit)
c_{ij}^{ot}	unit manufacturing cost of product *i* in PS *j* at overtime ($/unit)
c_{ijt}^{h}	unit inventory carrying cost of product *i* in PS *j* at the end of period *t* at the manufacturing echelon ($/unit)
c_{jt}^{Hr}	unit hiring cost in PS *j* at period *t* ($/man-hour)
c_{jt}^{Fr}	unit firing cost in PS *j* at period *t* ($/man-hour)
h_{it}	unit inventory carrying cost of product *i* at period *t* at the retailing echelon ($/unit)

$repc_{ijt}$	unit replenishment cost at the retailing echelon for product *i* at period *t* ($/unit)

Parameters related to inventory levels and ordering

$SSIP_{ijt}^{\min}$	the safety stock inventory at warehouse of PS *j* for product *i* at period *t* (unit)
$SSIR_{it}^{\min}$	the safety stock inventory at warehouse of retailer for product *i* in period *t* (unit)
$Initialinv_i$	initial inventory of the *i*-th product at retailing echelon (unit)
$Initialbor_i$	initial back order quantity of the *i*-th product at retailing echelon (unit)
$InitialinvP_{ij}$	initial inventory of the *i*-th product at production site *j* (unit)
rf_i	the percentage of the *i*-th product quantities to be ordered from credible manufacturers

Parameters of workforce levels

wl_{jt}	unit required labor of the *i*-th product at period *t* (man-hour/unit)
$wl_{jt}^{\max}$	the maximum labor level available to PS *j* at period t (man-hour)
$wl_{jt}^{\min}$	the minimum labor level available to PS *j* at period *t* (man-hour)

Parameters regarding machine and warehousing capacity

CWP_{ijt}	unit required warehouse space of the *i*-th product in PS *j* at period t (ft^2/unit)
CWR_{it}	unit required warehouse space of the *i*-th product at period *t* at the retailing echelon (ft^2/unit)
$CapPL_{jt}^{\max}$	the maximum warehouse space of manufacturing echelon available in PS *j* at period *t* (ft^2)
$CapR_t^{\max}$	the maximum warehouse space of retailer available at period *t* (ft^2)
$MCap_{jt}^{\max}$	the maximum machine capacity available at period *t* (machine-hour)
β_j	the fraction of machine capacity available for overtime use in PS *j* at each period
η_{jt}	unit required machine usage of PS *j* at period *t* (machine-hour/unit)

3.2.2. Decision variables

BR_{it}	backorder level of the *i*-th product in period *t* (unit)
x_{ijt}	regular time production of the *i*-th product at period *t* in production site *j* (unit)
y_{ijt}	amount of product *i* manufactured at PS *j* in overtime of period *t* (unit)
IP_{ijt}	inventory of product *i* at the end of period *t* at PS *j* (unit)
IR_{it}	inventory of product *i* in the retailing echelon at period *t* (unit)
Hr_{jt}	quantity of workforce hired in PS *j* at period *t* (man-hour)
Fr_{jt}	quantity of workers laid off in PS *j* at period *t* (man-hour)
P_{kit}	the selling price of the *i*-th product in MS *k* at period t ($/unit)
O_{ijt}	the order quantity of the *i*-th product in PS *j* at period *t* (unit)
M_{kt}	marketing expenditure per unit in MS *k* at period *t* ($/unit)
q_{it}	quality level of product *i* from the customers' point of view at period *t*

3.2.3. Assumptions

Based on the above characteristics of the considered APP problem, the mathematical model herein is developed on the following assumptions.

- Due to the paramount importance in channel-based price differentiation in real-world retailing, focal companies tend to segment the customers based on their demand sensitivity. Therefore, each market is divided into n_{ms} market segments and each segment is price differentiated based on the willingness of customers who are attributed by the retailer to that particular market segment. Braouezec (2016) and Braouezec (2012) highlighted that, in the monopoly literature, it is assumed that the underlying set of customers has already been divided in different segments[5].
- Power demand functions have been well accepted in the pricing and production management literature (e.g. Samadi et al., 2013; Sadjadi et al., 2010; Abad, 1988; Kim and Lee, 1998; Sadjadi et al., 2012; Arcelus and Srinivasan, 1987). We assume that the demand response curve is an increasing power function of the product quality and marketing expenditure. Naturally, this curve is also a decreasing power function of price. This kind of demand function formulates the customer behavior regarding quality, selling price and marketing expenditure.
- The production system at the manufacturing sites is aggregated into a capacitated single stage system i.e. considering the bottleneck stage (Torabi and Hassini, 2009).
- The forecasted demand over a particular period can be either satisfied or backordered, but the backorder must be fulfilled in the subsequent period (Entezaminia et al., 2016).
- The values of all parameters are certain over the planning horizon of the next T (Ghasemy Yaghin, 2018).
- There are limitations on budget, space and workforce available during the planning horizon. Actual workforce levels in regular time and overtime, production, warehouse space and advertising costs cannot exceed their respective maximum levels in each period.
- The unit shipping cost of each product from the manufacturing sites to retailer is involved in the replenishment costs.
- The retailing echelon faces multiple sales channels.
- Market segmentations are incomplete and demand leakage occurs from high-priced to lowest-priced segments (i.e. between MSs) (Zhang et al., 2010). In real-world situations, customers naturally try to find a way to switch to *the lowest priced segment*. Hence, all other market segments have demand leakage relationship with market segment n_{ms} (i.e. the lowest priced channel).
- The retailer orders a percentage of order quantities from credible factories with good reputation.

3.3. Model mathematical formulation

3.3.1. Proposed demand function

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[5]The interested reader may refer to Goldberg and Verboven (2001) and Einolf and Forgang (2006) to find real-world examples in which a monopolist operates in market segmentation environment.

Numerous demand functions have been proposed to estimate the future demands (e.g. Abad, 2003; Sadjadi et al., 2012; Ghasemy Yaghin et al., 2014). Here, we propose the following demand rate of the *i*-th product in MS *k* at period *t* by considering cannibalization between MSs. The proposed function is actually a generalization of Zhang et al. (2010) function which includes the geometric power function of price, marketing expenditure and product quality.

$$D_{kit}\left(P_{kit},P_{n_{ms}it},M_{kt},q_{it}\right)=\nu_{kit}P_{kit}^{-\alpha_{kit}}M_{kt}^{\gamma_k}q_{it}^{\theta_{it}}-LR(P_{kit}-P_{n_{ms}it}) \qquad k=1,..,n_{ms}-1 \qquad (1)$$

$$D_{n_{ms}it}\left(P_{1it},..,P_{n_{ms}it},M_{n_{ms}t},q_{it}\right)=\nu_{n_{ms}it}P_{n_{ms}it}^{-\alpha_{n_{ms}it}}M_{n_{ms}t}^{\gamma_{n_{ms}}}q_{it}^{\theta_{it}}+LR\sum_{k'=1}^{n_{ms}-1}(P_{k'it}-P_{n_{ms}it}) \qquad k=n_{ms} \qquad (2)$$

As stated before, one of the key parts of RM is to segment a single market into multiple submarkets/segments and then tailor different prices in each submarket. Since market segmentation which a firm operates in is imperfect, setting different prices for distinct market segments would cause the immigration of customers from one market segment to another. Without loss of generality, we index the retailer's market segments such that $P_1>P_2>...>P_{n_{ms}-1}>P_{n_{ms}}$. The market segment with the highest price is considered to be market segment 1, the market segment with the second highest price is considered to be market segment 2 and so on. Via this indexing, the last market segment (i.e. market segment n_{ms}) has the lowest price. In real-world situations, customers naturally try to find a way to switch to *the lowest priced segment*. This is because we assumed all other market segments have demand leakage relationship with market segment n_{ms} (i.e. the lowest priced channel). To clarify, Figure 1 shows the conceptual representation of the proposed demand function in two segments.

Figure 1 to be inserted here

3.3.2. Objective function

The total profit of the system is equal to the revenue minus total costs of the two-echelon supply chain. Therefore, the mathematical formulation of the total profit is as follows:

$$\begin{aligned}
MaxTPS^{Org} &= \sum_{k=1}^{n_{ms}-1}\sum_{i=1}^{n_g}\sum_{t=1}^{T_{ph}}\left(\nu_{kit}P_{kit}^{1-\alpha_{kit}}M_{kt}^{\gamma_k}q_{it}^{\theta_{it}}-LR\times P_{kit}^2+LR\times P_{kit}P_{n_{ms}it}\right)\\
&+\sum_{i=1}^{n_g}\sum_{t=1}^{T_{ph}}\left(\nu_{n_{ms}it}P_{n_{ms}it}^{1-\alpha_{kit}}M_{n_{ms}t}^{\gamma_{n_{ms}}}q_{it}^{\theta_{it}}+LR\left(\sum_{k'=1}^{n_{ms}-1}P_{k'it}P_{n_{ms}it}-(n_{ms}-1)P_{n_{ms}it}^2\right)\right)\\
&-\sum_{k=1}^{n_{ms}-1}\sum_{i=1}^{n_g}\sum_{t=1}^{T_{ph}}\left(\nu_{kit}P_{kit}^{-\alpha_{kit}}M_{kt}^{1+\gamma_k}q_{it}^{\theta_{it}}-LR\left(P_{kit}M_{kt}-P_{n_{ms}it}M_{kt}\right)\right)\\
&-\sum_{i=1}^{n_g}\sum_{t=1}^{T_{ph}}\left(\nu_{n_{ms}it}P_{n_{ms}it}^{-\alpha_{n_{ms}it}}M_{n_{ms}t}^{1+\gamma_{n_{ms}}}q_{it}^{\theta_{it}}+LR\left(\sum_{k'=1}^{n_{ms}-1}P_{k'it}M_{n_{ms}t}-(n_{ms}-1)P_{n_{ms}it}M_{n_{ms}t}\right)\right)\\
&-\sum_{i=1}^{ng}\sum_{j=1}^{n_s}\sum_{t=1}^{T_{ph}}c_{ij}^{rt}q_{it}^{\theta_{it}}x_{ijt}-\sum_{i=1}^{ng}\sum_{j=1}^{n_s}\sum_{t=1}^{T_{ph}}c_{ij}^{ot}q_{it}^{\theta_{it}}y_{ijt}-\sum_{i=1}^{ng}\sum_{j=1}^{n_s}\sum_{t=1}^{T_{ph}}c_{ijt}^{h}IP_{ijt}
\end{aligned}$$

$$-\sum_{j=1}^{n_s}\sum_{t=1}^{T_{ph}} c_{jt}^{Hr} Hr_{jt} - \sum_{j=1}^{n_s}\sum_{t=1}^{T_{ph}} c_{jt}^{Fr} Fr_{jt} - \sum_{i=1}^{n_g}\sum_{t=1}^{T_{ph}} h_{it} IR_{it} - \sum_{i=1}^{n_g}\sum_{j=1}^{n_s}\sum_{t=1}^{T_{ph}} repc_{ijt} O_{ijt} \quad (3)$$

Objective function (3) represents the difference between the revenue generated by selling the products in channels and the total cost. Equation (3) includes 11 mathematical terms. Terms (1-2) calculate the sum of $P_{kit} D_{kit}$ over all products in all segments in all periods. Terms (3-4) formulate the total marketing costs. Term 5 is the regular time production cost. Similarly, the overtime production cost is computed by Term 6. Term 7 represents the inventory cost of the finished products at sites. Hiring and firing costs are considered by Terms (8-9), respectively. Term 10 includes the holding cost at the retailing echelon.

3.3.3. Model constraints

The objective function is subject to the following constraints and equilibrium conditions:

$$Initialinv_i - Initialbor_i + \sum_{j=1}^{J} O_{ij1} - IR_{i1} + BR_{i1} = \sum_{k=1}^{n_{ms}} D_{ki1} \qquad \forall i \quad (4)$$

$$IR_{i,t-1} - BR_{i,t-1} + \sum_{j=1}^{J} O_{ijt} - IR_{it} + BR_{it} = \sum_{k=1}^{n_{ms}} D_{kit} \qquad t = 2,..,T, \forall i \quad (5)$$

$$IR_{it} \geq SSIR_{it}^{\min} \qquad \forall i,t \quad (6)$$

$$InitialinvP_{ij} + x_{ij1} + y_{ij1} - IP_{ij1} = O_{ij1} \qquad \forall i,j \quad (7)$$

$$IP_{ij,t-1} + x_{ijt} + y_{ijt} - IP_{ijt} = O_{ijt} \qquad t = 2,..,T, \forall i,j \quad (8)$$

$$IP_{ijt} \geq SSIP_{ijt}^{\min} \qquad \forall i,j,t \quad (9)$$

$$BR_{it} \leq BR_{it}^{\max} \qquad \forall i,t \quad (10)$$

$$\sum_{i=1}^{n_g} Wl_{i,t-1}\left(x_{ij,t-1} + y_{ij,t-1}\right) + Hr_{jt} - Fr_{jt} = \sum_{i=1}^{n_g} Wl_{it}\left(x_{ijt} + y_{ijt}\right) \qquad t = 2,..,T, \forall j \quad (11)$$

$$\sum_{i=1}^{n_g} Wl_{it}\left(x_{ijt} + y_{ijt}\right) \leq Wl_{jt}^{\max} \qquad \forall j,t \quad (12)$$

$$Hr_{jt}.Fr_{jt} = 0, \qquad \forall j,t \quad (13)$$

$$\sum_{i=1}^{n_g} CWP_{ijt} IP_{ijt} \le CapPl_{jt}^{\max} \qquad \forall j,t \qquad (14)$$

$$\sum_{i=1}^{n_g} CWR_{it} IR_{it} \le CapR_t^{\max} \qquad \forall t \qquad (15)$$

$$\sum_{i=1}^{n_g} \eta_{jt} \left(x_{ijt} + y_{ijt} \right) \le MCap_{jt}^{\max} \qquad \forall j,t \qquad (16)$$

$$\sum_{i=1}^{n_g} \eta_{jt}\, y_{ijt} \le \beta_j MCap_{jt}^{\max} \qquad \forall j,t \qquad (17)$$

$$\sum_{k=1}^{n_{ms}} \sum_{i=1}^{n_g} \sum_{t=1}^{T_{ph}} M_{kt} \Big[\nu_{kit} P_{kit}^{-\alpha_{kit}} M_{kt}^{\gamma_k} q_{it}^{\theta_{it}} - LR(P_{kit} - P_{Kit}) + \nu_{n_{ms}it} P_{n_{ms}it}^{-\alpha_{n_{ms}it}} M_{n_{ms}t}^{\gamma_{n_{ms}}} q_{it}^{\theta_{it}}$$

$$+ LR \sum_{k=1}^{n_{ms}-1} (P_{kit} - P_{n_{ms}it}) \Big] \le BUD^{\max} \qquad (18)$$

$$O_{ijt} \le MCap_{jt}^{\max} \qquad \forall i,j,t \qquad (19)$$

$$D_{kit} \ge 0 \qquad \forall k,i,t \qquad (20)$$

$$\sum_{j=1}^{n_s} rf_i O_{ijt} = \sum_{j \in BQ} \left(x_{ijt} + y_{ijt} \right) \qquad \forall i,t \qquad (21)$$

$$0 < q_{it} \le 1 \qquad \forall i,t \qquad (22)$$

$$BR_{it}, IP_{ijt}, IR_{ijt}, Hr_{jt}, Fr_{jt}, O_{ijt} \ge 0\ ,\ x_{ijt}, y_{ijt}, \mathrm{P}_{kit}, M_{kt} > 0 \qquad \forall i,j,k,t \qquad (23)$$

Constraints (4) and (5) are the balanced equations for the products in retailing echelon at the end of period 1 and period $t=2,..,T$, respectively. Constraint (6) provides safety stock levels at the retailing echelon. Constraints (7) and (8) are inventory and production balance equations at PS *j* for the *i*-th products at the end of period 1 and period $t=2,..,T$. Minimum safety stock limitations at the manufacturing echelon are imposed by constraint (9). Constraint (10) limits the shortage of products to the maximum allowed level.

Constraint (11) guarantees that the available workforce in each period is equal to the workforce in the previous period plus any changes in the workforce level during the current period. Constraint (12) expresses the upper bounds of labor levels at the PS *j*. Constraint (13) emphasizes that either hiring or firing is allowed in a particular period. The capacity restrictions for the warehouse's space of the retailer and PSs and machine capacity in PS *j* at period *t* are presented via constraints (15-17).

Constraint (18) reveals the limitation on the total budget available to marketing expenditures. The capacity constraint of PS *j* at each period for ordering quantities of the *i*-th product from retailing echelon is considered by inequality (19). Constraint (20) reflects the fact that the demand rate is non-negative. Ordering preferences of the *i*-th product at period *t* are taken into account by constraint (21). There is a fraction parameter used in this constraint, rf_i, in which the value of this fraction is determined by the retailer. By constraint (21), ordering from credible manufacturers with good reputation is ensured regarding this fraction. Constraint (22) shows that the quality level of the product from the customers' point of view cannot exceed 1.

Finally, Equations (23) demonstrate the 'variables' constrains enforcing non-negativity restriction for all decision variables. In the presence of the objective function and Constraints (4-5) and (18), the proposed model is a highly nonlinear model with some geometric terms.

4. The proposed solution methodology: Generalized geometric programming

The optimization model at hand is actually a non-convex non-linear programming (N-NLP) formulation (henceforth, we try to minimize the $TPS^{Org(1)} = -TPS^{Org}$ instead) with some geometric expressions. Generally speaking, geometric programming (GP) is an approach to solve algebraic non-linear optimization problems. Originally developed by Duffin et al. (1967), GP with posynomials provides a powerful method for studying many problems in optimal engineering design. Notably, the GP technique is efficiently capable of tackling this type of global optimization approach by properly selecting power transformation (Duffin et al., 1967; Boyd et al., 2007). Its intriguing structural properties as well as its elegant theoretical basis have caused a number of beneficial applications and the development of numerous useful results (e.g. Liu, 2009; Samadi et al., 2013).

Generalized geometric programming (GGP) is the class of optimization problems where the objective function and constraints are signomials. Importantly, (posynomial) GP fails to find the optimal solutions with the presence of signomial terms. This is so because GGP might have multiple minima. When (posynomial) GP formulations suffer from the presence of negatively signed monomials in the models for important applications, GGP has been studied by many researchers (e.g. Blau and Wilde, 1969; Floudas, 2000; Tseng et al., 2015).

Unlike GP problems in inventory control studies (e.g. Mandal et al., 2006; Sadjadi et al., 2012 and references therein), our mathematical model is a GGP one. This kind of formulation leads to a global optimization problem that is difficult to solve. The mathematical model at hand can be written as the difference of two posynomials. We embed the idea of Floudas (2000) based upon the convexification strategy i.e. transforming the non-convex signomial terms for convexification based on certain power functions or an exponential function because of its computational efficiency. However, our optimization problem is modeled accurately by mixtures of using signomials and more general types of algebraic functions. Hence, an analytical algorithm is tailored to search the solution space in an effort to determine the optimal policies. In fact, the proposed approach is based on modified GP approaches. In particular, the complicated mathematical optimization problem is converted into an equivalent convexified version by GP approaches and transformation techniques. The resultant model can be routinely solved via existing optimization solvers like MINOS or IPOPT in GAMS in order to find local optimum solutions.

4.1. Difference of two convex functions transformation

As we can see from the objective function and non-linear constraints, the proposed model is not a pure GP with posynomial terms. Actually, it includes some posynomials and also other types of nonlinear terms. Initially, all the terms in the objective and constraint functions of the resultant model are recognized and partitioned into the following classes: non-convex geometric terms, convex nonlinear and linear. We try to transform non-convex GP terms into equivalent convex expressions. As represented by Floudas (2000), the GPP problem can be formulated as the following nonlinear optimization problem:

$$\min_{x} G_0(x) = G_0^+(x) - G_0^-(x)$$

Subject to.

$$G_j(x) = G_j^+(x) - G_j^-(x) \le 0, \; j = 1,..,M$$

$$x_i \ge 0, \qquad i = 1,..,N \tag{24}$$

Where $G_j^+(x) = \sum_{k \in K_j^+} c_{jk} \prod_{i=1}^{N} x_i^{\alpha_{ijk}}, \qquad j = 0,..,M$

$$G_j^-(x) = \sum_{k \in K_j^-} c_{jk} \prod_{i=1}^{N} x_i^{\alpha_{ijk}}, \qquad j = 0,..,M$$

In the aforementioned optimization problem, $x = (x_1,..,x_N)$ is a vector of positive variables; $G_j^+(x), G_j^-(x)$'s are positive posynomial functions; α_{ijk} are arbitrary real constants and c_{jk} are the positive coefficients. Finally, K_j^+, K_j^- contain the positive/negative monomials that form the posynomial functions $G_j^+(x), G_j^-(x)$, respectively. Formulation (24) is a highly non-linear problem with a non-convex objective function and/or constraints (Floudas, 2000; Gounaris and Floudas, 2008). By applying the transformation $x_i = \exp(z_i)$**,** $i = 1,..,N$ on Problem (24), we obtain the following difference of two convex functions programming.

$$DC: \min_{x} G_0(z) = G_0^+(z) - G_0^-(z)$$

Subject to.

$$G_j(z) = G_j^+(z) - G_j^-(z) \le 0, \; j = 1,..,M$$

$$z_i^L \le z_i \le z_i^U, \qquad i = 1,..,N \tag{25}$$

Where $G_j^+(z) = \sum_{k \in K_j^+} c_{jk} \exp\left(\sum_{i=1}^{N} \alpha_{ijk} z_i\right), \qquad j = 0,..,M$

$$G_j^-(z) = \sum_{k \in K_j^-} c_{jk} \exp\left(\sum_{i=1}^{N} \alpha_{ijk} z_i\right), \qquad j = 0,..,M$$

in which all functions $G_j^+(z), G_j^-(z)$ that are positive linear combinations of convex functions are convex as well. As mentioned by Floudas (2000), a lower bound on the solution of problem

(DC) can be obtained by solving a convex relaxation of the original problem (DC). Every concave function $-G_j^-(z)$ can be underestimated with a linear function $-L_j^-(z),\quad j=0,..,M$. Consequently, this transformation results in the following relaxed convex programming problem whose solution provides a lower bound on the solution of (DC).

$$\min_x \ G_0^{Conv}(z)=G_0^+(z)-L_0^-(z)$$

Subject to.

$$G_j^{Conv}(z)=G_j^+(z)-L_j^-(z)\le 0,\ j=1,..,M$$

$$z_i^L \le z_i \le z_i^U,\qquad i=1,..,N \tag{26}$$

Where $G_j^+(z)=\sum_{k\in K_j^+} c_{jk}\exp\left(\sum_{i=1}^{N}\alpha_{ijk}z_i\right),\qquad j=0,..,M$

$$L_j^-(z)=\sum_{k\in K_j^-} c_{jk}\left(A_{jk}+B_{jk}\left(\sum_{i=1}^{N}\alpha_{ijk}z_i\right)\right),\qquad j=0,..,M$$

And $A_{jk}=\dfrac{Y_{jk}^U\exp(Y_{jk}^L)-Y_{jk}^L\exp(Y_{jk}^U)}{Y_{jk}^U-Y_{jk}^L}$

$$B_{jk}=\frac{\exp(Y_{jk}^U)-\exp(Y_{jk}^L)}{Y_{jk}^U-Y_{jk}^L}$$

$$Y_{jk}^U=\sum_{i=1}^{N}\max(\alpha_{ijk}z_i^L,\alpha_{ijk}z_i^U)$$

$$Y_{jk}^L=\sum_{i=1}^{N}\min(\alpha_{ijk}z_i^L,\alpha_{ijk}z_i^U)$$

Floudas (2000) analyzed the quality of this convex lower bounding by examining the tightness of the underestimation based on the maximum separation. The interested reader can refer to Floudas (2000) for more details.

4.2. Constraints and the objective transformation process

For non-convex terms that appeared in the objective function and constraints, we represent the geometric expression of our model by the following GGP classifications. It should be noted that these terms are obtained in $TPS^{Org(1)}$ after multiplication of TPS^{Org} by subtraction. Moreover, there are some linear expressions in the original model; it is apparent that no transformation strategy is needed for these terms.

$G_1^+=P_{kit}^{-\alpha_{kit}}M_{kt}^{\gamma_k}q_{it}^{\theta_{it}}$	Posynomial
$G_2^+=P_{kit}^{-\alpha_{kit}}M_{kt}^{1+\gamma_k}q_{it}^{\theta_{it}}$	Posynomial
$G_3^+=P_{n_{ms}it}^{-\alpha_{kit}}M_{n_{ms}t}^{1+\gamma_{n_{ms}}}q_{it}^{\theta_{it}}$	Posynomial
$G_4^-=P_{kit}^{1-\alpha_{kit}}M_{kt}^{\gamma_k}q_{it}^{\theta_{it}}$	Non-posynomial (negative monomial)
$G_5^-=P_{n_{ms}it}^{1-\alpha_{n_{ms}it}}M_{n_{ms}t}^{\gamma_{n_{ms}}}q_{it}^{\theta_{it}}$	Non-posynomial (negative monomial)
$G_6^+=q_{it}^{\theta_{it}}x_{ijt}$	Posynomial
$G_7^+=q_{it}^{\theta_{it}}y_{ijt}$	Posynomial

After the exponential transformations, $P_{kit}=\exp(P'_{kit})$, $M_{kt}=\exp(M'_{kt})$, $q_{it}=\exp(q_{it})$, $x_{ijt}=\exp(x'_{ijt})$ and $y_{ijt}=\exp(y'_{ijt})$, the following relations would be obtained.

$$G_1^{Conv}=\exp\left(-\alpha_{kit}P'_{kit}+\gamma_k M'_{kt}+\theta_{it}q'_{it}\right) \quad (27)$$

$$G_2^{Conv}=\exp\left(-\alpha_{kit}P'_{kit}+(1+\gamma_k)M'_{kt}+\theta_{it}q'_{it}\right) \quad (28)$$

$$G_3^{Conv}=\exp\left(-\alpha_{n_{ms}it}P'_{n_{ms}it}+(1+\gamma_{n_{ms}})M'_{n_{ms}t}+\theta_{it}q'_{it}\right) \quad (29)$$

$$G_4^{Conv}=\exp\left((1-\alpha_{kit})P'_{kit}+\gamma_k M'_{kt}+\theta_{it}q'_{it}\right) \quad (30)$$

$$G_5^{Conv}=\exp\left((1-\alpha_{n_{ms}it})P'_{n_{ms}it}+\gamma_{n_{ms}}M'_{n_{ms}t}+\theta_{it}q'_{it}\right) \quad (31)$$

$$G_6^{Conv}=\exp\left(\theta_{it}q'_{it}+x'_{ijt}\right) \quad (32)$$

$$G_7^{Conv}=\exp\left(\theta_{it}q'_{it}+y'_{ijt}\right) \quad (33)$$

To linearize the negative monomial terms, the following relations would be calculated:

$$Y_{4,kit}^{L}=\min\left((1-\alpha_{kit})P'^{L}_{kit},(1-\alpha_{kit})P'^{U}_{kit}\right)+\min\left(\gamma_k M'^{L}_{kt},\gamma_k M'^{U}_{kt}\right)+\min\left(\theta_{it}q'^{L}_{it},\theta_{it}q'^{U}_{it}\right) \quad (34)$$

$$Y_{4,kit}^{U}=\max\left((1-\alpha_{kit})P'^{L}_{kit},(1-\alpha_{kit})P'^{U}_{kit}\right)+\max\left(\gamma_k M'^{L}_{kt},\gamma_k M'^{U}_{kt}\right)+\max\left(\theta_{it}q'^{L}_{it},\theta_{it}q'^{U}_{it}\right) \quad (35)$$

$$A_{4,kit}=\frac{Y_{4,kit}^{U}\exp(Y_{4,kit}^{L})-Y_{4,kit}^{L}\exp(Y_{4,kit}^{U})}{Y_{4,kit}^{U}-Y_{4,kit}^{L}} \quad (36)$$

$$B_{4,kit}=\frac{\exp(Y_{4,kit}^{U})-\exp(Y_{4,kit}^{L})}{Y_{4,kit}^{U}-Y_{4,kit}^{L}} \quad (37)$$

$$Y_{5,n_{ms}it}^{L}=\min\left((1-\alpha_{n_{ms}it})P'^{L}_{n_{ms}it},(1-\alpha_{n_{ms}it})P'^{U}_{n_{ms}it}\right)+\min\left(\gamma_{n_{ms}}M'^{L}_{n_{ms}t},\gamma_{n_{ms}}M'^{U}_{n_{ms}t}\right)+\min\left(\theta_{it}q'^{L}_{it},\theta_{it}q'^{U}_{it}\right) \quad (38)$$

$$Y_{5,n_{ms}it}^{U}=\max\left((1-\alpha_{n_{ms}it})P'^{L}_{n_{ms}it},(1-\alpha_{n_{ms}it})P'^{U}_{n_{ms}it}\right)+\max\left(\gamma_{n_{ms}}M'^{L}_{n_{ms}t},\gamma_{n_{ms}}M'^{U}_{n_{ms}t}\right)+\max\left(\theta_{it}q'^{L}_{it},\theta_{it}q'^{U}_{it}\right) \quad (39)$$

$$A_{5,n_{ms}it}=\frac{Y_{5,n_{ms}it}^{U}\exp(Y_{5,n_{ms}it}^{L})-Y_{5,n_{ms}it}^{L}\exp(Y_{5,n_{ms}it}^{U})}{Y_{5,n_{ms}it}^{U}-Y_{5,n_{ms}it}^{L}} \quad (40)$$

$$B_{5,n_{ms}it}=\frac{\exp(Y_{5,n_{ms}it}^{U})-\exp(Y_{5,n_{ms}it}^{L})}{Y_{5,n_{ms}it}^{U}-Y_{5,n_{ms}it}^{L}} \quad (41)$$

The following linearization transformation is obtained:

$$L_{4,kit}^{-}(z)=A_{4,kit}+B_{4,kit}\left[(1-\alpha_{kit})P'_{kit}+\gamma_k M'_{kt}+\theta_{it}q'_{it}\right]$$

$$L_{5,\mathrm{n}_{ms}it}^{-}(z)=A_{5,\mathrm{n}_{ms}it}+B_{5,\mathrm{n}_{ms}it}\left[(1-\alpha_{\mathrm{n}_{ms}it})P'_{\mathrm{n}_{ms}it}+\gamma_{\mathrm{n}_{ms}}M'_{\mathrm{n}_{ms}t}+\theta_{it}q'_{it}\right]$$

In this manner, the transformation could be done on G_5^{-}. In an almost similar way, the constraints with posynomial and/or non-posynomial expressions can be coped with. According to the transformation described previously, the equivalent formulation of the mathematical model can be derived as follows.

$$
\begin{aligned}
&Max\,TPS^{Org} = \\
&\sum_{k=1}^{n_{ms}-1}\sum_{i=1}^{n_g}\sum_{t=1}^{T_{ph}}\Big(\nu_{kit}\,(A_{4,kit}+B_{4,kit}\left[(1-\alpha_{kit})P'_{kit}+\gamma_k M'_{kt}+\theta_{it}q'_{it}\right])-LR\times P_{kit}^2 \\
&+LR\times P_{kit}P_{n_{ms}it}\Big)+\sum_{i=1}^{n_g}\sum_{t=1}^{T_{ph}}(\nu_{n_{ms}it}\,(A_{5,\mathrm{n}_{ms}\,it}+B_{5,\mathrm{n}_{ms}\,it}\left[(1-\alpha_{\mathrm{n}_{ms}\,it})P'_{\mathrm{n}_{ms}\,it}+\gamma_{\mathrm{n}_{ms}}M'_{\mathrm{n}_{ms}\,t}+\theta_{it}q'_{it}\right]) \\
&+LR\left(\sum_{k'=1}^{n_{ms}-1}P_{k'it}P_{n_{ms}it}-(n_{ms}-1)P_{n_{ms}it}^2\right)) \\
&-\sum_{k=1}^{n_{ms}-1}\sum_{i=1}^{n_g}\sum_{t=1}^{T_{ph}}\Big(\nu_{kit}\exp\big(-\alpha_{n_{ms}it}P'_{n_{ms}it}+(1+\gamma_{n_{ms}})M'_{n_{ms}t}+\theta_{it}q'_{it}\big)-LR\left(P_{kit}M_{kt}-P_{n_{ms}it}M_{kt}\right)\Big) \\
&-\sum_{i=1}^{n_g}\sum_{t=1}^{T_{ph}}\Big(\nu_{n_{ms}it}\exp\big(-\alpha_{kit}P'_{kit}+(1+\gamma_k)M'_{kt}+\theta_{it}q'_{it}\big) \\
&+LR\left(\sum_{k'=1}^{n_{ms}-1}P_{k'it}M_{n_{ms}t}-(n_{ms}-1)P_{n_{ms}it}M_{n_{ms}t}\right))-\sum_{i=1}^{ng}\sum_{j=1}^{n_s}\sum_{t=1}^{T_{ph}}c_{ij}^{rt}\exp\big(\theta_{it}q'_{it}+x'_{ijt}\big)- \\
&-\sum_{i=1}^{ng}\sum_{j=1}^{n_s}\sum_{t=1}^{T_{ph}}c_{ij}^{ot}\exp\big(\theta_{it}q'_{it}+y'_{ijt}\big)-\sum_{i=1}^{ng}\sum_{j=1}^{n_s}\sum_{t=1}^{T_{ph}}c_{ijt}^{h}IP_{ijt}-\sum_{j=1}^{n_s}\sum_{t=1}^{T_{ph}}c_{jt}^{Hr}Hr_{jt} \\
&-\sum_{j=1}^{n_s}\sum_{t=1}^{T_{ph}}c_{jt}^{Fr}Fr_{jt}-\sum_{i=1}^{n_g}\sum_{t=1}^{T_{ph}}h_{it}IR_{it}-\sum_{i=1}^{n_g}\sum_{j=1}^{n_s}\sum_{t=1}^{T_{ph}}repc_{ijt}O_{ijt}
\end{aligned}
\tag{42}
$$

Subject to

$$
\begin{aligned}
&\sum_{k=1}^{n_{ms}}\sum_{i=1}^{n_g}\sum_{t=1}^{T_{ph}}M_{kt}\Big[\nu_{kit}\exp\big(-\alpha_{kit}P'_{kit}+\gamma_k M'_{kt}+\theta_{it}q'_{it}\big)-LR(P_{kit}^{Sl}-P_{Kit}^{Sl}) \\
&+\nu_{n_{ms}it}\exp\big(-\alpha_{n_{ms}it}P'_{n_{ms}it}+\gamma_{n_{ms}}M'_{n_{ms}t}+\theta_{it}q'_{it}\big)+LR\sum_{k=1}^{n_{ms}-1}(P_{kit}-P_{n_{ms}it})\Big]\le BUD^{\max}
\end{aligned}
\tag{43}
$$

$$
x_{ijt}=\exp\left(x'_{ijt}\right),\,y_{ijt}=\exp\left(y'_{ijt}\right),\,P_{kit}=\exp\left(P'_{kit}\right) \tag{44}
$$

$$
M_{kt}=\exp\left(M'_{kt}\right),\,q_{it}=\exp\left(q'_{it}\right) \tag{45}
$$

(4-17), (19-22).

This procedure allows us not only to convexify and underestimate every generalized signomial constraint but also to convexify the entire non-convex geometric programming model.

4.3. Solution procedure

Our proposed solution procedure based on GGP is described as follows**:**

Step 1: Formulate the considered APP problem,

Step 2: Compute and obtain the difference of two convex functions programming,

Step 3: Generate convex underestimator for each posynomial term with exponential transformation based on Section 4.1,
Step 4: Linearize the concave terms regarding Sections 4.1 and 4.2,
Step 5: Obtain the equivalent convexified model and locally solve the equivalent convexified-NLP by a solver,
Step 6: Obtain the lower bound for TPS^{Org} and compute inverse transformation and find original decision variables.

5. A case study: Clothing supply chain

As mentioned by Nayak and Padhye (2015), the apparel industry stands out as one of the most globalized industries in the world. It is a supply driven commodity chain led by a combination of players; each plays an important role in a network of supply chains that spans from fibers to yarn, to fabrics, to accessories, to garments, and finally to marketing. Therefore, the main purpose of the numerical experiments is to investigate the applicability and appropriateness of the proposed framework using a simulated instance from a typical clothing supply chain. In fact, the necessity of a proposed approach towards differential marketing and supply chain production planning is shown using the application.

5.1. Setup

A case study inspired by a clothing supply chain is presented for demonstrating the validity and practicality of the proposed model and solution procedure. The supply chain is supposed to produce six products from three production factories to fulfill the demand of three submarkets of a retailing echelon over a six-period planning horizon. In the clothing market, the retailing echelon needs to ensure that the manufacturing reputation preference can be fulfilled. Ordering preferences, i.e. ordering to credible manufacturers with good reputation, play an important role in the production planning process of textile products (Leong et al., 2003). Due to confidentiality, most of the input data are randomly generated. However, the generation process is done so that they will be close to the real data available in the considered supply chain. Table 2 summarizes the source of random data generation. In this table, term ‘‘U’’ and “N” implicate the uniform and the normal distribution, respectively. Without loss of generality and just to simplify the imperfect segmentation, we consider leakage behavior to the lowest price MS, i.e. n_{ms} -th market. Other relevant data regarding this case are as follows:

- Immigration behavior is taken into account by a constant proportion of leakage.
- Retailing echelon experiences price-dependent power function demand, which is affected by marketing expenditure and quality of textile products.
- The first manufacturer has a good reputation in the market, i.e. $BQ \in \{1\}$. The clothing retail store desires to order a percentage of orders from a credible manufacturer with good reputation.

- Quantity of workforce hired does not represent the number of new workers coming on, but rather the quantity of new, trained workers. These workers may have been hired in previous periods.

Table 2 to be inserted here

5.2. Result and discussion

The proposed model has been applied to the preceding data set to find the optimal production-pricing plan. The resultant convexified mathematical programming formulation has been modeled within the general algebraic modelling system (GAMS) environment and solved by IPOPT/MINOS solver. The given problem is solved on a Pentium IV running at 2 GHz on the Windows 7 operating system. Obtained solutions of APP are shown in Tables 3-10.

Table 3 to be inserted here

Table 4 to be inserted here

It can be seen from Table 3 that clothing retailing echelon can tailor different prices for the same product by multiple demand classes. It is expectedly interesting that prices in the third market segments are the lowest because of imperfect segmentation. This will lead to movement between market segments, which is known as demand leakage. Table 3 provides an insight into the result, showing that the product prices of some periods are set higher. It is caused by significant demand variability and elasticity during these periods. While differentiating customers by their willingness-to-pay into multiple, the influences of the leakage behavior and moving between MSs cause considerable changes in the prices levels. The reported data indicate that our model and solution procedure work properly. In more general terms, as the difference between prices of segments grows, the customers are tempted to find a way to pay lower prices. Thus, more often than not, it decreases the utility of differential pricing. In other words, the outcome of such a behavior results in an unpleasant and undesirable segmentation. In the real world, this is much more likely to occur than the situation where customers buy the clothing products from their assigned segment. As correctly suggested by Zhang et al. (2010), considering appropriate "fences", as a device to preserve market segmentation, can lead to limited spillover between segments and then economic benefit of multiple demand classes.

It should be noted that alongside the determination of the price quantities, it is certainly possible to generate marketing expenditures and product quality from the customer point of view. The results in Tables 4 and 5 reveal product quality at each period and marketing expenditures that the retailer should spend at some periods in order to influence the customer demand. As can be seen from Table 5, marketing expenditures are reduced in some periods when demand exceeds the capacity. Obviously, the retailer is not allowed to expend at some periods (see blank cells) due to maximum budgeting cost of marketing activities that has to be guaranteed. Moreover, regarding regulations and enforcement in some countries, the retailer is faced with a marketing budget limitation. Thus, adoption of these issues causes no expense on marketing efforts in the aforementioned periods.

Table 5 to be inserted here

Table 6 to be inserted here

Production plan for each plant in regular time can be seen in Table 6. As shown in Table 6, all product types are manufactured in all factories at each period. The items to be supplied from the credible manufacturer are determined regarding the retailer preferences. What is more, manufacturer 1 produces all the items at about all periods.

Table 7 to be inserted here

Table 8 to be inserted here

Table 9 to be inserted here

Ordering from each plant can be seen in Table 9. As shown in Table 9, the retailer orders all product types from all factories at all periods. Findings suggest that all production sites in the considered supply chain are involved with the retailers.

Table 10 to be inserted here

We can observe from Table 10 that the model results reveal a greater hiring workforce level compared to firing over the planning horizon; and there is no hiring for the latest period. Due to the demand variability of clothing products, changing the workforce levels over a planning horizon is mandatory. The results from Table 10 approve such a phenomenon. Moreover, firing and employee layoff is under control based on the considered set of data.

As it is seen from Table 10, due to production quantities, in successive periods 3,4 and 5, worker-hours reduce and as a result hiring increases and then we have hiring workforces and from period 5 it begins to stop and we only have layoffs.

5.3. Performance evaluation

To evaluate the performance of the proposed analytical solution algorithm, twenty random data sets are generated. Then, their convex lower bounding values are compared with the solutions of the original non-convex optimization problem at hand. We use the global solver of GAMS (BARON) to obtain the optimal solutions in limited-size instances. Table 11 shows the solution of the analytical approach and the best solution of the solver. To estimate the deviation of our algorithms with the lower bound, Equation (46) is used. F_{GP} indicates the corresponding profit of our proposed approach, and F_{Sl} refers to the results given by the global solver for the non-convex problem.

$$Dev = \frac{F_{GP} - F_{Sl}}{F_{Sl}} \tag{46}$$

Table 11 to be inserted here

This comparison indicates the effectiveness of the proposed solution approach (GGP) to cope with the non-convexities. Based on Table 11, it can be stated that the tightness of the lower bound is about 11.92% in comparison to the global solutions.

5.4. Sensitivity analysis

This sub-section discusses analyzing the sensitivity of the decision model to some parameters. The parameters that are changed are as follows:

BUD^{max} : The maximum marketing cost available to marketing activities of the focal company. It varies from -0.50% to 0.50%

LR : The leakage fraction between demand classes. This parameter is changed as 0.15, 0.25, 0.4, 0.6, 0.8 and 0.9

ν_{kit} : Scaling constant of price-dependent demand

α_{kit} : Price elasticity to demand term.

It is worth noting that the scaling constant and price elasticity of the first product in channel 1 are considered to analyze the changes on the total profit.

Table 12 to be inserted here

Table 13 to be inserted here

Tables (12-13) illustrate the results of the sensitivity analysis. It is obvious that these parameters have significant influences on the total profit. Maximum budget available to marketing activities indicates that the more the marketing budget increases, the more total profit will increase. In other words, the total profit of the system is sensitive to changes of maximum available marketing cost. The results of Table (12) also reveal that setting different prices for submarkets could augment the immigration of customers from high-priced markets to lowest-priced ones. As can be seen from Table (12), leakage rate provides negative impact on total profit. The more the leakage rate increases, the less the total profit is. This leads to the conclusion that leakage rate is observed to be more influential in multi-channel retailing. This finding also coincides with the literature (Zhang et al., 2010; Ghasemy Yaghin, 2018).

Investigating the effects of changing price elasticity, price elasticity in the proposed geometric demand response curve has negative impact on profitability (See Table 13). As Table (13) shows, an increase in the scaling constant of price-dependent demand declares that the total profit increases. Summarizing our findings, we conclude that variation of marketing and demand parameters influence the total profit.

5.5. Managerial implications

The formulated model incorporates the decisions on ordering, multi-site APP, differential pricing and quality-dependent demand in a two-echelon supply chain with manufacturing ordering preferences. It is taken for granted that it is difficult to make all these decisions because they are closely interrelated. To cope with this, an integrated multi-site aggregate production-pricing planning model along with marketing issues is developed. The valuable structure of the proposed mathematical model with geometric terms results in a novel solution procedure.

The research findings indicate a positive relation between the scaling constant of price-dependent demand and total profit rate and a negative impact of wide price gaps on profitability. We draw several insights from the above numerical studies.

i) Our numerical study shows the leakage behavior has a negative impact on the total profit. Under the price differentian, as the customers' motivation is enhanced to switch between segments, the total profit is decreased (see Table 12). Hence, the results of our numerical study clearly imply that retailers who engage in the channel-based retailing should be enabled to deal with demand leakage between market segments. As price gaps grow, the customers are tempted to find a way to pay lower prices, thus, utility of differential pricing is decreased. Consequently, the key condition for the successful price differentiation is to apply appropriate 'fences' to prevent the customer movement.

ii) Investigation of the price sensitivity provides a clear understanding about our price-response function. This parameter, known as *price elasticity,* measures the sensitivity of demand to price. We found that the price elasticity changes the total profit in the presence of customer response curve with geometric terms. This is intuitive: for a higher elasticity parameter, the supply chain has to expect less profit. Furthermore, scaling constant has a great impact on the total profit (See Table 13). It is worthwhile to note that precisely knowing the demand structure is an important prerequisite for practitioners in the profit maximization of the supply chain. Hence, supply chain planning managers should be rather certain about the reliability and accuracy of the data that are used to forecast price elasticity and scaling constant over the midterm planning horizon in all market segments.

iii) While firms allocate limited financial resources for marketing activities, our numerical study shows an increased marketing budget can increase profitability. This finding provides practitioners with a valuable insight to assign higher investment in marketing-oriented activities instead of following the fixed overall marketing budget strategy. In the integrated production-marketing environment, this finding helps the managers to improve the profit by increasing the marketing budget.

6. Concluding remarks

In most logistical planning situations, the issues of ordering, price-setting and aggregate planning are settled by negotiation between manufacturers, retailers and marketers. Such a process often results in a near optimal or optimal policy for one party of the chain; in some cases, non-optimal policies for all parties. In this paper, an integrated multi-site supply chain aggregate production-marketing planning model is developed through the lens of the revenue management that incorporates some realistic features. First, imperfect market segmentation and cannibalization behavior are involved in the demand management of the retailing echelon that makes a broader application scope for considering different types of customers. Second, we consider a novel price and quality-dependent customer demand with geometric terms in developing the model, which permits a proper recognition of critical parameters of the customers' demand in marketing analysis especially for the garment products. Third, a generalized geometric programming-based solution procedure is taken into account for a highly non-linear mathematical model in order to find the equivalent convexified formulation.

The aggregate production-marketing planning model has not come to an end and the path is still open for researchers to study some combinations of differential pricing of integrated supply chains and global optimization approaches to take advantage of them simultaneously.

From a supply chain modelling perspective, we propose two possible directions. First one is to explore the current warehousing strategy of cross-docking that involves movement of material directly from the receiving dock to the shipping dock with minimum dwell time. Second, more complicated supply chain decisions such as procurement, manufacturing and/or multi-echelon distribution systems with sustainability consideration could be involved similar to that of Sarkar et al. (2016). From a solution methodology perspective, it will be interesting to analyze more computational studies that would help improve the solution of global optimization programs through the successive refinement of a convex relaxation and the subsequent solution of a series of nonlinear convex optimization problems. Although the proposed algorithm works satisfactorily with the numerical study, more computational studies will be required to test the efficiency of the proposed technique, especially for large-scale problems. Generally, GGP is a global optimization algorithm with great potentials for power functions. Furthermore, the application of other methods adapting the meta-heuristics algorithms is another area recommended for future research.

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Appendix.

Figure A.1, proposed by Tang (2010), presents the conceptual framework for coordination between marketing and production.

Put Figure A.1 here

Tables

Table 1: Review of some existing models.

Reference	Multiple manuf. plants	SC structure P: Production, R: Retailing, SR: Sales Region, C: Sales Channel, D: Distribution	Reverse logistics	Pricing	Ordering preferences	Quality issues	Marketing aspects	Multiple demand classes	Optimization approach
Aucamp (1986)		P		√			√		LP
Feiring and Mak (1995)		P		√					LP
Leung et al. (2003)	√	P			√				LP
Leung et al. (2006)	√	P							LP
Yenradee and Sarvi (2007)		P		√			√		LP
Torabi and Hassini (2008)	√	S-P-D							LP
Leung and Chan (2009)	√	P							LP
Yenradee and Piyamanothorn (2011)		P					√		LP
Al-e-hashem et al. (2011)	√	S-P-SR							MONLP
Ghasemy Yaghin et al. (2012)		P-R		√			√		MONLP
Torabi and Moghaddam (2012)	√	P-SR							MOLP
Gholamian et al. (2015)	√	S-P- SR							MOLP
Entezaminia et al. (2016)	√	S-P- SR	√						MOLP
Gholamian et al. (2016)	√	S-P- SR							MOLP
Ghasemy Yaghin (2018)	√	P-SR		√			√	√	NLP
This paper	√	P-R-C		√	√	√	√	√	GP

Table 2: The data set for the supply chain under consideration.

Parameter	Corresponding random distribution	Parameter	Corresponding random distribution
$v_{1it}, t \le 3$	$U(3,5)\times 10^6$	θ_{it}	$U(0.4,0.6)$
$v_{1it}, t \ge 4$	$U(5,7)\times 10^6$	h_{it}	$N(12,9)$
$v_{2it}, t \le 3$	$U(2,3)\times 10^6$	$repc_{ijt}, j \le 2$	$N(115,16)$
$v_{2it}, t \ge 4$	$U(4,5)\times 10^6$	$repc_{ijt}, j \ge 3$	$N(200,16)$
$v_{3it}, t \le 3$	$U(0.7,2)\times 10^6$	$C^{h}_{ijt}, j \le 2$	$N(6,9)$
$v_{3it}, t \ge 4$	$U(2,4)\times 10^6$	$C^{h}_{ijt}, j \ge 3$	$N(12,9)$
α_{1it}	$N(3,0.01)$	$C^{Hr}_{jt}, t \le 3$	$N(70,25)$
α_{2it}	$N(2.5,0.01)$	$C^{Hr}_{jt}, t \ge 4$	$N(90,16)$
α_{3it}	$N(2.2,0.01)$	$C^{Fr}_{jt}, t \le 3$	$N(45,25)$
γ_k	$U(0.3,0.6)$	$C^{Fr}_{jt}, t \ge 4$	$N(25,25)$
LR	$U(0.2,0.4)$	$Initialinv_i$	$U(20,25)$
C^{rt}_{ij}	$N(250,81)$	$InitialinP_{ij}$	$U(16,20)$
C^{ot}_{ij}	$N(400,196)$	$Initialbor_i$	$U(0,10)$
$SSIR^{\min}_{it}, t \le 3$	$U(0,12)$	Wl_{jt}	$U(8,12)$
$SSIR^{\min}_{it}, t \ge 4$	$U(0,6)$	$Wl^{\max}_{jt}$	$U(400,480)$
$SSIP^{\min}_{ijt}, t \le 3$	$U(8,12)$	$Cappl^{\max}_{jt}, j \in BQ$	$U(420,475)$
$SSIP^{\min}_{ijt}, t \ge 4$	$U(10,16)$	$Cappl^{\max}_{jt}, j \notin BQ$	$U(300,325)$
$\eta_{jt}, j \in BQ$	$U(0.1,0.15)$	$CWP_{ijt}, i \le 3$	$N(1.5,0.25)$
$\eta_{jt}, j \notin BQ$	$U(0.13,0.2)$	$BUD^{\max}$	$N(1500,900)$
β_j	$U(0.5,0.65)$	rf_i	$U(0.3,0.5)$
$CapR^{\max}_t$	$U(325,375)$	CWR_{it}	$Average(CWP_{ijt})\times U(0.95,1.05)$

Table 3: Market prices to be set (in $).

Submarkets	Products	Periods					
		1	2	3	4	5	6
1	1	40.44	50.04	52.605	61.43	62.25	57.01
	2	54.98	53.26	59.94	53.68	61.22	65.94
	3	64.50	59.83	51.68	61.47	63.00	64.45
	4	67.44	57.22	58.16	68.58	69.64	63.40

	5	64.08	59.45	59.98	64.21	59.08	62.79
	6	64.86	58.84	59.03	62.70	57.64	67.55
2	1	33.08	39.14	40.48	43.30	53.34	43.30
	2	43.81	41.85	46.12	49.26	50.70	49.62
	3	43.20	43.84	43.44	48.92	48.51	43.75
	4	42.75	48.00	47.00	49.76	48.67	45.93
	5	44.46	48.71	50.52	56.81	48.16	47.02
	6	46.58	42.06	45.57	45.84	49.08	43.36
3	1	25.08	33.32	39.02	37.02	43.58	38.69
	2	31.87	36.48	32.64	41.28	38.95	38.51
	3	31.729	30.56	32.93	35.31	38.92	37.00
	4	32.48	34.62	39.93	47.52	38.95	37.56
	5	33.73	30.29	33.66	39.33	34.15	33.86
	6	30.14	32.73	33.79	37.19	39.42	30.91

Table 4: Quality level of the product from the customers point of view.

	Periods					
Products	1	2	3	4	5	6
1	0.898	0.969	0.915	0.909	0.93	0.929
2	0.909	0.905	0.928	0.912	0.925	0.926
3	0.931	0.944	0.963	0.911	0.919	0.926
4	0.909	0.951	0.927	0.908	0.925	0.927
5	0.909	0.910	0.924	0.912	0.926	0.928
6	0.911	0.895	0.880	0.913	0.91	0.924

Table 5: Marketing expenditure quantities (in $).

	Periods					
Submarkets	1	2	3	4	5	6
1	11.941	10.142	17.4	5.723	8.237	8.706
2	10.809		9.507	11.012	14.625	
3	12.25	14.21		8.118		

Table 6: Regular time production quantities assigned in each plant to produce the products (in unit).

Production sites	Products	Periods					
		1	2	3	4	5	6
1	1	18.737	36.40	50.96	11.88	12.17	
	2	13.025	37.0	52.19	14.45	16.31	35.27
	3	33.001	16.210	14.096	9. 942		7.78
	4	9.579	15.632	16.681		16.372	12.91
	5	26.385	13.628		23586	24.400	15.986
	6		19.332	16.373	8.687	27.891	
2	1	13.977	12.872	11.025	12.766	10.872	
	2	14.889	14.204	8.587		13.071	
	3	12.441	14.148	23.558		13.102	8.296
	4		17.537		15.339		
	5	12.749				14.458	17.999
	6	18.327	4.508	22.905		18.928	8.439
3	1	28.892	27.915		28.663	20.867	27.244
	2		24.306	15.343	23.078	14.006	
	3	23.628	24.371		9.332	6.373	17.187
	4		14.409	12.445			13.769
	5		24.692	15.311	26.88	13.957	19.356
	6	23.96	24.71	25.41		35.32	

Table 7: Inventory level of production sites (in unit).

Production sites	Products	Periods				
		1	2	3	4	5
1	1				2.337	
	2	28.470	35.81	42.379		
	3			3.159	5.494	7.397
	4	32.857	37.379	34.117		
	5					
	6			1.108	4.313	
2	1		6.516		2.366	
	2	31.447	39.104	40.420		
	3		2.538		4.3699	7.358
	4		7.657	13.23		
	5	26.6	2.925			
	6				4.144	
3	1				2.215	
	2	20.846	10.505	6.524		
	3		2.913	6.103	6.169	8.481
	4	34.550	26.154	26.892		1.997
	5					
	6			5.099	5.989	

Table 8: Inventory level of retailer at each period (in unit).

Products	Periods					
	1	2	3	4	5	6
1	14.619	12.539	8.802	12.478	9.403	13.491
2	10.431	11.154	12.72	11.463	12.633	10.760
3	12.463	12.331	10.781	10.028	12.169	13.052
4	12.596	9.183	12.491	12.968	13.554	11.382
5	11.534	10.194	12.759	13.598	11.842	14.552
6	9.563	9.778	13.590	9.508	9.562	10.446

Table 9: Quantities to be ordered at retailing echelon (in unit).

Products	Production sites	Periods					
		1	2	3	4	5	6
1	1	10.446	28.459	12.62	7.77	8.204	10.54
	2	10.241	18.606	13.154	9.197	8.608	10.181
	3	12.318	17.587	13.925	8.741	8.283	8.365
2	1	8.176	19.227	12.654	8.894	7.784	10.598
	2	9.818	19.968	13.329	8.854	7.90	10.325
	3	11.462	22.217	12.088	7.579	7.790	8.536
3	1	10.823	20.447	9.79	7.776	7.764	10.561
	2	10.995	20.247	11.516	9.239	8.006	9.788
	3	12.285	18.027	12.860	8.782	8.271	8.542
4	1	8.876	22.166	10.003	7.264	7.541	9.824
	2	8.768	22.235	10.808	10.291	8.186	10.532
	3	11.598	21.698	11.521	7.781	7.892	8.915
5	1	9.176	17.125	9.211	9.576	9.520	10.588
	2	9.228	20.854	11.818	9.752	7.576	11.883
	3	10.868	22.103	11.820	8.409	8.151	9.319
6	1	7.963	23.559	11.307	7.172	7.633	9.233
	2	9.640	23.419	12.040	8.191	7.676	9.487
	3	13.863	18.687	11.879	8.052	8.079	8.10

Table 10: Hiring and firing levels (in man-hour).

	H_{jt}						F_{jt}					
	1	2	3	4	5	6	1	2	3	4	5	6
1			5.361		5.806		1.421	17.05		2.881		
2				18.30	9.705			6.88	13.11			12.777
3			10.046	18.73	9.870			13.87				8.451

Table 11: Comparison of the obtained computational results.

Data set	Objective value			Data set	Objective value		
	Proposed Algorithm	Best solution	Gap		Proposed Algorithm	Best solution	Gap
1	1.1646	1.0068	0.156734	11	5.1128	4.5755	0.11743
2	1.827	1.603	0.139738	12	8.5638	7.7053	0.11141
3	2.208	1.9775	0.116561	13	1.1199	0.98728	0.13432
4	1.1368	1.0013	0.135324	14	2.4059	2.178	0.10463
5	0.09628	0.0872	0.104128	15	0.88357	0.7701	0.14734
6	1.2522	1.1215	0.11654	16	0.079181	0.07029	0.12649
7	1.8668	1.6845	0.108222	17	3.9528	3.5804	0.10401
8	3.1132	2.8189	0.104402	18	5.7293	5.0683	0.13041
9	6.915	6.2944	0.098596	19	1.9217	1.7366	0.10658
10	1.0228	0.9132	0.120018	20	6.5291	5.9283	0.10134

Table 12: Sensitivity analysis on maximum marketing budget and leakage rate.

LR	Changes in $BUD^{\max}$ (%)				
	–0.50	–0.25	0.25	0.50	0.75
0.15	5.1509	5.7692	6.3598	7.0127	7.2918
0.25	4.9336	5.4715	6.1562	6.7146	6.7773
0.4	4.4128	5.2361	6.4060	6.5760	6.6125
0.6	4.02049	4.8875	6.0209	6.1256	6.3953
0.8	3.7869	4.5881	5.7642	5.7703	5.9043
0.9	3.5201	4.0881	5.1583	5.3021	5.6117

Table 13: Sensitivity analysis on scaling constant of price-dependent demand and price elasticity.

α_{kit} (%)	Changes in ν_{kit} (%)				
	–0.50	–0.25	0.25	0.50	0.75
–0.5	6.1677	6.6201	7.1115	7.3290	7.6114
–0.25	5.7285	6.3019	6.8908	6.9825	7.3691
0.25	5.2409	5.9114	6.3989	6.5519	6.9109
0.5	4.8912	5.2943	6.0917	6.2104	6.4519